

# ***Product Reject Analysis using Weibull Distribution Model***

## ***A Case Study in Automotive Parts Manufacturing Industry***

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**Abstract**— Recently, single statistical tools analyses are used to analyze product reject. Bar chart, line chart and histogram are the most common tools used. However, simple statistical tool analysis could only analyze single point of view and obtain single information in each analysis. This study presents the application of Weibull Distribution model in analyzing characteristics pattern of product reject data. Weibull Distribution is an advanced and flexible distribution model that commonly used for processing and interpreting life data. Therefore, the main objective of this study is to determine root cause of the reject product and level of production performance by providing statistical value and distribution plotting graph from the analysis using Weibull Distribution model. The analysis of data shows there are seven defect types occurred during the production of product. This study focus on a critical type of defect which contributes to the highest reject rate. The possible root cause will be determined from the result of the analysis.

**Keywords**—Weibull Distribution; product reject; case study; maintenance; reliability

### **I. INTRODUCTION**

Product reject can be define as the unreasonable danger on product. It is also being said by [1] that reject product is related to safety. Lack of quantification assessment and attention for the reject on product should be change. Zero reject products never exist in actual manufacturing. Therefore, defective produce should be standard reject and does not put the user on danger. Product reject can be divided into four which are design reject, manufacturing reject, lack of warning and instruction reject. Mostly, reject are come from manufacturing process that are possibly caused by low quality of raw material and operation mistake. Therefore, tracking the product reject during inspection is important to prevent the harm or problem during use.

In manufacturing line consist of variety process begin from raw material until finished goods. Within the process, understanding material properties and inspection activity are important due to quality reason. Even the task is critically done, product reject still can exist [2]. Material, design and processing method contributes to the causing reject products. Accordingly, finding root cause of failure mode is vital to prevent the continuity of the stated problem.

Product quality affect the cost, profit and company's image. From engineering perspective view, problem of product reject could be analyze by using statistical methods, visual inspection, and various engineering techniques. As an example, a defect product happen in fabric manufacturing uses FDDS tools to analyze the real time of fabric reject and gray level co-occurrence matrix (GLCM) for textural of the fabric problem [3]. All techniques used improved the inspection process, and the reject could be easily detected.

Data can be analyzed by data presentation or static. Data presentation is the display data such as Pareto, Histogram, line chart bar chart and pie chart. Static data presents information by number or value from some calculation. Statistical analysis basic involve of mean, standard deviation, coefficient of variation, confidence limit and propagation of error. Statistic is the method to measure and analysis process either used in manufacturing or nonmanufacturing process. The objectives of statistical analysis are to improve the quality of process, achieve stability, and solve processing problems [4]. Several types of statistical analysis tool are control chart, cause and effect diagram, check sheet, pareto chart and reject concentration diagrams.

Currently, product reject is analyse by using simple statistical tool analysis. Nonetheless, it is limited to analyze single point of view. The user will only getting a single information for each analysis [5]. The information of the result

analysis will not be discussed when using simple statistical tool. This problem may cause the analysis result not clear and lack of information.

Weibull Distribution model is one of advance statistical tool which attract more attention in statistical analyses. Weibull Distribution is a flexible distribution model technique for processing and interpreting life data. Flexibility of Weibull Distribution was proven with wide variety of possible failure rate curve shapes. The result can be related with failure rate and bathtub curve concept. Bathtub curve is the concept of the life time of machine. Therefore, Weibull Distribution model often applied in reliability method.

Two-parameter Weibull Distribution function was used to analyse statistically the fatigue life results of composite samples. The result obtained was a fatigue life distribution diagrams for Glass-fiber reinforced polyester composite (GFRP) composites which represent the reliability percentage corresponding to any life (cycle) or stress amplitude[6].

Then, a two-component and three-component mixture of two and three parameter Weibull distribution was also used to analyze time to failure data. Parameter were estimated using maximum likelihood estimation method and the shapes of the density or hazard functions were used to determine the suitability of mixture Weibull distribution to represent the real failure times data for the cases[7].

In this paper, we present the application of Weibull Distribution model to analyze product reject problems based on reject rate data. A case study in an automotive part in manufacturing industry is used for the Weibull Distribution model application. From the statistical point of view, product reject data will be analyze and the result will be discuss in detail to find their possible root cause.

## II. CHARACTERISTICS OF WEIBULL DISTRIBUTION MODEL

Weibull Distribution model is one of the most useful probability distribution in reliability. In order to compute reliabilities, there are four related probability functions that being used. Reliability function, cumulative distribution function (CDF), probability density function (PDF) and hazard rate function are commonly used to calculate reliabilities but offer four different perspectives[8].

Reliability is defined as probability that a system or component will function over some time period  $t$ . When  $T \geq 0$ , reliability function,  $R(t)$ , can be expressed as

$$R(t) = \Pr\{T \geq t\} \quad (1)$$

For a given value of  $t$ ,  $R(t)$  is the probability that the time to failure is greater than or equal to  $t$ . Cumulative distribution function (CDF),  $F(t)$ , of the failure distribution also can be define as;

$$F(t) = 1 - R(t) = \Pr\{T < t\} \quad (2)$$

Probability density function (PDF),  $f(t)$ , is then defined by

$$f(t) = dF(t)/dt = -dR(t)/dt \quad (3)$$

From (3), the mean time to failure (MTTF) can be defined by

$$\begin{aligned} \text{MTTF} &= \int_0^\infty t f(t) dt = \int_0^\infty t [-dR(t)/dt] dt \\ \text{MTTF} &= \int_0^\infty R(t) dt \end{aligned} \quad (4)$$

Meanwhile, hazard rate function also called the failure rate function,  $\lambda(t)$ , provides an instantaneous (at time  $t$ ) rate of failure.

$$\Pr\{t \leq T \leq t + \Delta t\} = R(t) - R(t + \Delta t)$$

Then,  $[R(t) - R(t + \Delta t)] / R(t) \Delta t$  is the conditional probability of failure per unit of time (failure rate).

$$\lambda(t) = \lim_{\Delta t \rightarrow 0} \{-[R(t + \Delta t) - R(t)] / \Delta t\} \cdot 1/R(t)$$

$$\lambda(t) = -dR(t)/dt \cdot 1/R(t)$$

$$\lambda(t) = f(t) / R(t) \quad (5)$$

From (5), Weibull failure distribution is characterized of the form  $\lambda(t) = at^b$  which is a power function.

when  $\alpha > 0$ ,  $\beta > 0$ ,  $t \geq 0$

$$\lambda(t) = (\beta/\alpha) (t/\alpha)^{\beta-1} \quad (6)$$

where  $\alpha$  (theta) is called the scale parameter and  $\beta$  (beta) is called the shape parameter.

It can be seen that when  $\beta < 1$ , the failure rate function is decreasing (DFR) and when  $\beta > 1$ , it is increasing (IFR). In the case of  $\beta = 1$ , we have the well-known exponential distribution which has a constant failure rate (CFR). [9]. In the case of product reject analysis,  $\beta$  refer as number of reject rate and  $\alpha$  is refer as mean of product reject.

## III. CASE STUDY

A case study was carried out at a manufacturing industry located at northern area in peninsular Malaysia. The main product produced by this company is gasket. Gasket is widely used as an interface with the engine between the two parts. These products work to prevent leakage between the connections. Production of gasket involves basic types of part based on customer requirements.

For the whole gasket production consists of five parts with its own code. There are L Crankcase Cover (11191-KFM), R Crankcase Cover (11394-KYZ), Cylinder (12191-KFL), Cylinder Head (12251-KFL), and Oil Pump Body (15119-178).

Gaskets are sensitive products because of its nature. It is susceptible to damage because it is thin. There are seven types of defect that can cause it to be rejected which are break, poor

surface finish, poor cutting process, stretching, pollution, design unrequited, and marking. Among all the reject types, 'Break' types of reject have the highest reject rate with total of 3302 per year. Only several parts of gasket suffer from this type of reject which are part L Crankcase Cover (11191-KFM), Cylinder (12191-KFL), and Oil Pump Body (15119-178). Thus, analysis will only focus on selected part only which is part L Crankcase Cover (11191-KFM) as shown in Figure 1.



Fig. 1. Part L Crankcase Cover (11191-KFM)

This company recorded the raw data for number of reject per month and number of product produce per month of certain failure mode. As one of the critical failure modes is 'Break' type, then it will be the focus data. Before start the analysis, there is a correlation identification test is done to determine the relationship between total reject versus total production. Correlation test is important to identify either the reject data can be analyze using Weibull Distribution or not. The scatter plots that do not have correlation only can proceed to analyze by using Weibull Distribution model.

#### IV. RESULT AND DISCUSSION

Based from the case study, the part that will be analyzed by the Weibull Distribution model is part no 11191-KFM. Data taken for this part are from two years back data which in year 2012 and 2013. The comparison between the two years data can identify the trend characteristic of the reject part.

As mention before, the Weibull Distribution model can analyze data that only have no correlation on the scatter plot. The correlation identification test proves that there is no correlation between the number of product produce and the number of reject for the selected month. The initial analysis towards the obtained result then can be proceeded to use the renewal model which is Weibull Distribution model. Table 1 showed the analysis result from using Weibull Distribution for the selected part.

TABLE I. SUMMARIZE DATA OF THE WEIBULL DISTRIBUTION MODEL FOR PRODUCT REJECT BASED ON 'BREAK' TYPE MODE FOR YEAR 2012 AND 2013

YEAR	2012	2013
$\beta$	1.85135	1.31276
$\alpha$	35.7294	59.4192
CORRELATION	0.974	0.933
NO OF SAMPLE OF REJECT	10	6

Based from Table 1, the comparison result of 'Break' reject type for part 11191-KFM in between 2012 and 2013 can be seen clearly according to the value of  $\beta$  (beta) and  $\alpha$  (theta), correlation and number of sample of rejects.

Value of  $\beta$  or reject rate was related with consistency of the work. The  $\beta$  values in both year showed that the consistency is there.  $\beta$  value in 2012 is slightly higher than year 2013 which current year had reduce the reject rate from 1.85135 to 1.31276. Nevertheless, it is not significantly improved even though the value show that it is in decreasing failure rate (DFR).

However, the  $\beta$  value obtain does not reflect the mean of the product reject (value of  $\alpha$ ). This can be seen by the data of both years mean to double. The scale parameter noted as  $\alpha$  in year 2013 is higher than its previous year. Year 2012 shows number of  $\alpha$  is 35.7294 which represent almost 36 parts rejected averagely in each month. Meanwhile, in year 2013 shows an increment of 59.4192 which it up to 60 parts per month rejected averagely for the entire year. This result shows number of reject increased about up to 25% within a year.

Result shows that reject rate ( $\beta$ ) and scale parameter ( $\alpha$ ) are not moving together. The correlation both years are closer to positive one (+1). This is means that both year analysis results indicates fit with a positive slope. The linear relation of data is strength and relevant.

By using Weibull Distribution, the analysis result could identify inconsistency and abnormality practice happen in the production line of the products. The high reject rate related with inconsistency of working that possibly caused by the human error. Basically, human worker is attached to manual process which commonly related to lacked of flexibility in managing the order.

Manual work process that attached operator to complete the parts is process of removing waste and checking the product condition. Based from the information required by the company, the total number of workers in the department is five operators and been divided based on the types of product material. The operators are not allowed to handle other types of product that they had assigned, means that they are not be able to have rotational working schedule. This resulted in operator of fatigue and dullness. In year 2012, only one skillful worker is attached to handle the part to remove waste. The next year, the company has added one more worker without any working skill. The number of reject increased in the respective year because the assigned operator did not being provided with special training since he hired or having any necessary skills. Lack of skills to handle the tool to remove the waste may be one of the root causes. Wrong technique in handling material makes the part to easily break.

The environment during working also contributes to higher reject rate. For the working area is at high temperature and without any air-conditioner. Hot temperature may lead operators to easily become fatigue and uncomfortable to finish their job during eight hour of each shift.

Due to the root cause discovered, all of the problems should be avoided or at least reduces. As recommendation, continuous training could be done to the new worker with interesting allowances, and upgrade the working environment. To maintain the consistency level of worker, some working task or planned schedule should be rotated to comfort the operator as when they are on duty.

## V. CONCLUSION

This paper presented the Weibull Distribution model application for product reject analysis. Product reject data are analyzed from the statistical point of view and finding the possible root cause. Analysis had focus on one type of defect which contributes to the highest reject rate known as part L Crankcase Cover (11191-KFM). These types of reject which is break type have the highest reject rate with total of 3302 per year. From the analysis, the statistical value obtained could trace an abnormality practice occur in the production line. High reject rate related with the inconsistency of operator working ethics or their daily routine practice. Lack of skill of operator is expected as the main issue of causing the reject type for both stated year. This case study is relevant and significant to the application of the Weibull Distribution model where the abnormalities of the analysis result obtain could identify the problem when dealing with human task. This is one of the uniqueness of the Weibull Distribution model to be implemented in the maintenance area.

Weibull Distribution can be the alternative method to be use in order to analyze product reject analysis. It can represent hidden characteristic in the production processes. However, comprehensive analyses could be done for obtaining better result. As a future research, Weibull Distribution model can be

integrated with other advanced statistical tool such as FMECA when the data deals with verbal type to characterize systematically other defect mode.

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