Impact of Different Code Size on the Performance of 2-D Hybrid FCC-MDW code in OCDMA System

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***Abstract* - A new two dimensional (2-D) Wavelength/Time (W/T) scheme is proposed to enhance the capacity and performance of the system. This new 2-D Hybrid FCC-MDW code is utilizing Flexible Cross Correlation (FCC) and Modified Double Weight (MDW) codes which both codes capable to suppress multiple access interference (MAI) and reduced the PIIN. The numerical results reveal that the value of time spreading (spatial) code length, N is the major impact on the cardinality of the system. The performance of the system can be enhanced by exploiting the number of N rather than the value of wavelength encoding (spectral) code length, M.**

Index Terms* - *Optical Code Division Multiple Access, Modified Double Weight, Flexible Cross Correlation

I. Introduction

The past thirty years have seen increasingly rapid advances in the field of optical CDMA system [1]. The motivations of OCDMA being the most attractive multiple access techniques from a networking perspective are by following three potential [2]. At first, as compared to the spectral division of WDM, OCDMA offers a larger channel count. Next, asynchronous transmission simplifies medium access control (MAC) as compared to TDMA. Finally, the multirate services can be implemented by variable code length and code weight of the signature code simultaneously.There have been several studies reporting that performance and the cardinality of the system can be optimized in 2-D coding such as in [3],[4],[5],[6],[7]. The relationship between code length and system performance has been widely investigated such as in [3]. In addition, [8] mentioned that code length is an important feature of code and system design. Moreover, with a larger code length, the correlation properties among the codes can be improved. Consequently the system performance in terms of MAI, BER and throughput can be improved.

This paper provides new 2-D Hybrid FCC-MDW codes which combine the two different codeword to improve the system performance as well as the cardinality of the system. Utilizing MDW codes with fixed in-phase cross correlation and the flexibility of in-phase cross correlation in FCC code as well as the capability of both codes to eliminate MAI inspired us to evaluate the performance of these codes. From the analytical results of 2-D Hybrid FCC-MDW with different code size verified that the increment number of time spreading code length (spatial code length), N significantly affects the system performance compared to wavelength encoding code length (spectral code length), M.

The rest of this paper is organized as the construction and correlation property description of 2-D Hybrid FCC-MDW codes are detailed in Section II, performance analysis of 2-D Hybrid codes is provided in Section III, numerical results are shown and discussed in Section IV and finally, the conclusion of this paper in Section V.

II. 2D Hybrid FCC-MDW Code

Basically the construction of 2-D Hybrid FCC-MDW code is the combination of 1-D MDW code and 1-D FCC code. This proposed code denoted by ($M \times N$, w, λ_a, λ_c); M is the number of wavelengths, N is the temporal code length, w is code weight, λ_a and λ_c is auto-correlation and cross correlation respectively. Therefore, $M \times N$ is the code size of the 2-D Hybrid FCC-MDW.

A. 1-D MDW code

The weight of 1-D MDW code can be any even number that is greater than two as defined in [9].

As a family of DW code, MDW can also be represented by using the K x N matrix as shown in Fig.1.

$$\left|\begin{array}{c:c} A & B \\ \hdashline C & D \end{array}\right|$$

Fig.1 K x N matrix for 1-D MDW code

where the elements of A consists of $1 \times 3\sum_{j=1}^{\frac{w}{2}-1} j$ matrix of zeros, B is 1 x 3$\boldsymbol{n}$ matrix containing the basic matrix of [X_2] for every 3 columns, C is the basic code matrix for the next smaller weight, W = 2($\boldsymbol{n}$-1) and D is a matrix $\boldsymbol{n}$ x $\boldsymbol{n}$ consisting of basic matrix of [X_3] arranged as

$$\begin{vmatrix} 000 & 000 & [X_3] \\ 000 & [X_3] & 000 \\ [X_3] & 000 & 000 \end{vmatrix}$$

Two basic components in the basic matrix for MDW codes are code length $N_B = 3\sum_{j=1}^{\frac{W}{2}} j$ and number of user, $K_B = \frac{W}{2} + 1$ representing $K_B \times N_B$ where N_B is the column and K_B is the row. The example of MDW code with code length 9, weight 4 and an ideal in-phase cross-correlation denoted by (9, 4, 1) is shown in Table I.

TABLE I
1D MDW CODE SEQUENCES

(N,w,λ)	Code length	Number of user	Code weight	Code sequences
(9, 4, 1)	9	3	4	000011011 011000110 110110000

B. 1-D FCC code

As disclosed in [10] for a given *K* user, hamming-weight *w*, in-phase cross correlation λ_{max} and code length N, the set of the code can be represented by a KxN matrix A_K^w where the elements a_{ij} of A_K^w is binary [0,1] can be written as:

$$A_K^w = \left\{ a_{ij} = '0' \; or \; '1' for \begin{matrix} i = 1,2, \ldots K \\ j = 1,2, \ldots N \end{matrix} \right\} \quad (1)$$

Equation (1) is known as Tridiagonal Code Matrix and can be expressed as (2) which the rows A_1, A_2, A_k represent the K code words.

$$A_K^w = \begin{bmatrix} a_{11} & b_{12} & c_{13} & 0 & \cdots & 0 & 0 \\ 0 & d_{14} & a_{21} & b_{22} & \cdots & 0 & 0 \\ 0 & 0 & c_{23} & d_{24} & \cdots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & c_{i-1} & a_{KN} \end{bmatrix} = \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_K \end{bmatrix} \quad (2)$$

The *K* rows of the code matrix in (2) give the K OCDMA codes having flexible in phase cross-correlation, Hamming-weight *w* and minimum code length. One of the matrices binary sequences as shown in (2) whose the first *ith* row for the first K users can be expressed by (3) while the length *N* of code matrix or the length of the rows of KxN matrix has defined as (4).

$$A_i = \overbrace{0 \ldots\ldots 0}^{r(i-1)} \; \overbrace{1 \; 1 \ldots . 1}^{w} \; \overbrace{0 \ldots 0}^{r(K-i)} = \mathrm{N} \quad (3)$$

$$N = w\,K - \lambda_{max}(K - 1) \quad (4)$$

The Tridiagonal code matrix of the code with *K*= 4, *w*= 3, λ_{max} =1, is shown in Table II.

TABLE II
1-D FCC CODE SEQUENCES

(N,w,λ_{max})	Cord length	Number of users	Code weight	Code sequences
(9, 3, 1)	9	4	3	111000000 001110000 000011100 000000111

As above mentioned, the development of 2-D Hybrid FCC-MDW codes is the combination of two different codes i.e 1-D FCC and 1-D MDW codes. Let code sequences for 1-D MDW represented by X = { $x_0, x_1, \ldots\ldots x_{M-1}$} and Y = { $y_0, y_1, \ldots\ldots y_{N-1}$} represents 1-D FCC code sequences. Then, the 2-D Hybrid FCC-MDW can be generated by $A_{g,h} = Y_h^T X_g$ where $g \in (1,2,3,\ldots..,M-1)$ and $h \in (1,2,3,\ldots.., N-1)$. Y_h is the time spreading patterns while X_g is the wavelength encoding patterns. Table III shows some examples of 2-D Hybrid FCC-MDW code sequences for k1 = 4 and k2 = 2, where k1 and k2 are the code weights for X_g and Y_h respectively.

TABLE III
2-D HYBRID FCC-MDW CODE FOR K1 =4 AND K2=2 SEQUENCES

$A_{g,h}$	[000011011]	[011000110]	[110110000] X_g
1	000011011	011000110	110110000
1	000011011	011000110	110110000
0	000000000	000000000	000000000
0	000000000	000000000	000000000
1	000011011	011000110	110110000
1	000011011	011000110	110110000
Y_h			

Four characteristic matrices $A^{(d)}$, $d \in (0, 1, 2, 3)$ has been introduced in [3] to obtain the cross correlation property. Following the same assumption, $A^{(d)}$ for 2-D Hybrid FCC-MDW codes can be defined as:

$$A^{(0)} = Y^T X \quad (5)$$

$$A^{(1)} = Y^T \overline{X} \quad (6)$$

$$A^{(2)} = \overline{Y}^T X \quad (7)$$

$$A^{(3)} = \overline{Y}^T \overline{X} \quad (8)$$

Parameter $\overline{X}$ and $\overline{Y}$ are the complementary of *X* and *Y* respectively. The cross correlation of 2-D Hybrid FCC-MDW code $A^{(d)}$ and $A_{g,h}$ is expressed as

$$R^{(d)}(g,h) = \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} a_{ij}^{(d)} a_{(i+g)(j+h)} \quad (9)$$

where $a_{ij}^{(d)}$ is the (*i*, *j*) th of $A^{(d)}$ and $a_{(i+g)(j+h)}$ is the (*i*, *j*) th of $A_{g,h}$. Table IV illustrates the cross correlation between any two codes $A^{(d)}$ and $A_{g,h}$ of 2-D Hybrid FCC-MDW code generated from (9).

TABLE IV
CROSS CORRELATION OF 2-D HYBRID FCC-MDW CODE

$X_{g,h}$	$R^{(0)}(g,h)$	$R^{(1)}(g,h)$	$R^{(2)}(g,h)$	$R^{(3)}(g,h)$
$g=0,h=0$	$k1k2$	0	0	0
$g=0,h\neq 0$	$k1$	0	$k1$	0
$g\neq 0,h=0$	$k2$	$k2\,(k1-1)$	0	0
$g\neq 0,h\neq 0$	1	$k1-1$	1	$k1-1$

This property is very important in order to cancel the MAI and suppress the PIIN. The derivation of new correlation functions can be expressed as

$$R^{(0)}(g,h) - \frac{1}{(k1-1)}R^{(1)}(g,h) + \frac{1}{(k1-1)}R^{(3)}(g,h) - R^{(2)}(g,h) = \begin{cases} k_1 k_2, & \text{for } g=0 \text{ and } h=0 \\ 0, & \text{otherwise} \end{cases} \quad (10)$$

III. PERFORMANCE ANALYSIS

The correlation function values, BER and codeset cardinality are highly considered in order to analyze the performance of 2D Hybrid FCC-MDW code. The Gaussian approximation is used to calculate the bit error rate (BER). Subsequently, three types of noises are taken into account for calculating the BER including PIIN, shot noise as well as thermal noise in the photodiodes. The general form of the photocurrent noise emitted from the photodiodes can be expressed as follow:

$$\langle i^2\rangle = I^2 B\tau c + 2eIB + \frac{4K_bT_nB}{R_L} \quad (11)$$

where I is the average photocurrent output from the photodiode, B is the electrical bandwidth, τ_c is the coherence time of the light source, e is electron's charge, K_b is Boltzmann's constant, T_n is the absolute receiver noise temperature and R_L is the load resistance.

In addition, the following assumptions are made to simplify the analysis. Firstly, the output of broadband light source is ideally unpolarized and has flat spectrum over $\left[f_0 - \frac{\Delta f}{2}, f_0 + \frac{\Delta f}{2}\right]$ where f_0 and Δf are the central frequency and the bandwidth of the source. Secondly the spectral width of each spectral component is identical. Thirdly, every single user has equal received power and lastly, bit stream from different transmitter are synchronous. Based on the abovementioned assumption, the power spectral densities of the received optical signals can be written as [6];

$$r(f) = \frac{P_{sr}}{k2\Delta f}\sum_{w=1}^{W} d(w).\sum_{i=0}^{M-1}\sum_{j=0}^{N-1} a_{ij}(w) \times \left\{u\left[f - f_o - \frac{\Delta f}{2M}(-M+2i)\right] - u\left[f - f_o - \frac{\Delta f}{2M}(-M+2i+2)\right]\right\} \quad (12)$$

where P_{sr} the effective source power at the receiver, k2 is the code weight of the time spreading code sequence, W is the number of simultaneous active users, $d(w)$ is the data bit of the w th user, which can be "1" or "0", M is the code length of the wavelength encoding code sequence, N is the code length of the time spreading code sequence, $a_{ij}(w)$ represents an element of the w th user's code word while $u(f)$ is the unit step function.

By using cross correlation between codeword $A_{g,h}$ and $A_{0,0}^{(d)}$ as shown in Table IV, the equations of total output photocurrent, PIIN and shot noise can be determined. The total output photocurrent from the receiver can be expressed as:

$$I_r = \frac{\Re P_{sr} k1}{M} \quad (13)$$

where k_1 is the code weight of the wavelength encoding code sequence. Accordingly, the power of PIIN that exists in photocurrent of the receiver and under the circumstances that all of the users transmit bit "1" [3],[5], [6] can be written as:

$$< i_{PIIN}^2 > = \frac{B_r \Re^2 P_{sr}^2}{k_2^2 M\Delta f(MN-1)^2}\{[k1k2\,(MN-1) + k2\,(W-1)(M-1)]^2 + [k2\,(W-1)(M-1)]^2\} \quad (14)$$

In addition, by applying the same method as in (14) the power of shot noises from all photodiodes can be written as:

$$< i_{shot}^2 > = 2eB_r\frac{\Re P_{sr}}{Mk2}\left[k1k2 + 2k1\frac{(W-1)(N-1)}{(MN-1)} + 2k2\frac{(W-1)(M-1)}{(MN-1)} + 4\frac{(W-1)(M-1)(N-1)}{(MN-1)}\right] \quad (15)$$

Furthermore, by assuming that the probability of each user sending bit "1" is equal or ½ [6], the equation of PIIN and shot noise can be revised as (16) and (17) respectively.

$$< i_{PIIN}^2 > = \frac{B_r \Re^2 P_{sr}^2}{2k_2^2 M\Delta f(MN-1)^2}[k1k2\,(MN-1) + k2\,(W-1)(M-1)]^2 + [k2\,(W-1)(M-1)]^2 \quad (16)$$

$$< i_{shot}^2 > = \frac{eB_r \Re P_{sr}}{Mk2}\left[k1k2 + 2k1\frac{(W-1)(N-1)}{(MN-1)} + 2k2\frac{(W-1)(M-1)}{(MN-1)} + 4\frac{(W-1)(M-1)(N-1)}{(MN-1)}\right] \quad (17)$$

Moreover, the thermal noise can be written as:

$$< i_{thermal}^2 > = \frac{4K_b T_n B_r}{R_L} \quad (18)$$

Accordingly, by using the Eq. (13), (16), (17) and (18) the SNR at the receiver can be obtained as:

$$SNR = \frac{< I_r^2 >}{< i_{PIIN}^2 > + < i_{shot}^2 > + < i_{thermal}^2 >} \quad (19)$$

Therefore, the BER can then be estimated as:

$$BER = \frac{1}{2}\, erfc\left(\sqrt{\frac{SNR}{8}}\right) \quad (20)$$

IV. RESULTS AND DISCUSSION

The parameters used in our analysis are listed in Table V. The numerical results are presented in Fig.2 and Fig.3.

TABLE V
TYPICAL PARAMETERS USED IN THE CALCULATION

Parameters Used in Numerical Calculation	
PD quantum efficiency Spectral width of broadband light source	$\eta = 0.6$ $\Delta\lambda = 30$nm ($\Delta f = 3.75$ THz)
Operating wavelength	$\lambda_o = 1.55$ µm
Electrical bandwidth	B = 320 MHz
Data transmission rate	$R_b = 622$ Mbps
Receiver noise temperature	$T_n = 300$K
Receiver load resistor	$R_L = 1030\ \Omega$
Boltzmann's constant	$K_b = 1.38 \times 10^{-23}$ W/K/Hz
Electron charge Light velocity	$e = 1.60217646 \times 10^{-19}$ coulombs $C = 3 \times 10^8$ m/s

Fig.2 illustrated the BER against the number of simultaneous users with similar code size when the value of M is fixed at 23 and the effective source power, P_{sr} is fixed at -10dBm. The respective code sizes of 2-D Hybrid FCC-MDW are 161, 207 and 115 for (M = 23, N = 7), (M = 23, N = 9) and (M = 23, N = 5) respectively. From the data in Fig.2, it is apparent that when the value of M or wavelength encoding code length is fixed at 23 and the value of N is varied, there is a significant difference between the three groups. Fig.2 shows for N =7 the number of simultaneous users is 160, 210 and 120 for N =9 and N=5 respectively at the standard acceptable BER= 10^{-9}. These numerical results indicate that the lowest number of N (time spreading code length) gives the lower cardinality in the system. On the other hand, the higher value of N can accommodate the higher cardinality.

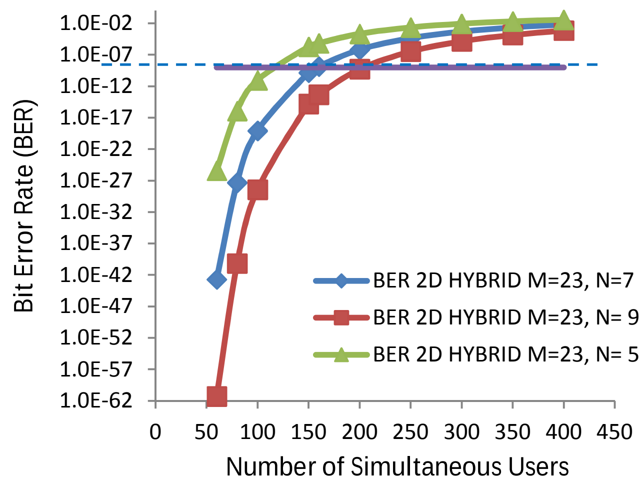

Fig.2 BER versus number of simultaneous users when the value of M is fixed.

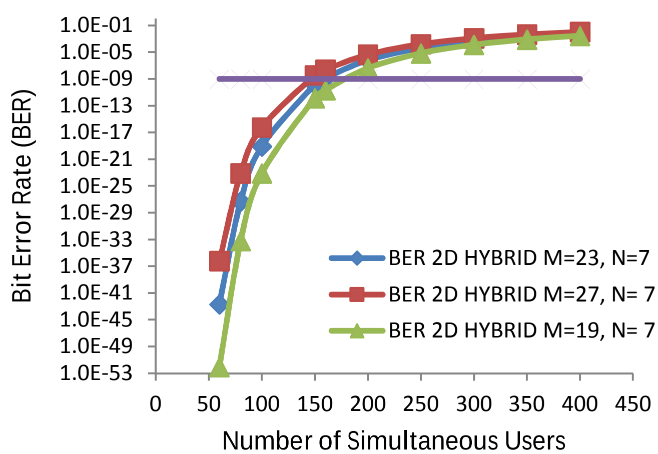

Fig.3 BER versus number of simultaneous users with the fixed value of N.

Fig. 3 discussed the BER versus the number of simultaneous users without changing the value of time spreading code length, N. As Fig.3 shows, there is only a small effect to the cardinality of the system. It can be clearly seen from the Fig.3, the varying value of M, makes a small different in the number of simultaneous users. The respective number of simultaneous users for the groups of code size (M = 23, N = 7), (M = 27, N =7) and (M = 19, N = 7) are 160, 150 and 180 respectively. It reveals that this finding is consistent with the findings of past studies by [3], which the improvement of the number of the chip time (N) has a significant effect compared to increasing the number of wavelengths.

V. CONCLUSION

The present study was designed to determine the effect of the code size of new 2D Hybrid FCC-MDW code which integrate the 1-D FCC code and 1-D MDW code as a time spreading and wavelength encoding respectively. The results of this investigation show that the time spreading code length, N plays the main role in order to enhance the cardinality of the system. In fact, a large number of N can accommodate the higher cardinality in the system at standard acceptable BER and vice versa when the number of wavelength code length, M is fixed. On the contrary, without changing the value of N and the values of M are varied, only a small effect on the number of simultaneous users can be obtained. Furthermore, these numerical results enhance our understanding of the relationship between the code size and the system performance.

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