Modeling of Spring-Mass-Damper Piezoelectric Equivalent Circuit for Multi-cantilever in Harvesting wide-band vibration sources

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***Abstract*— Spring-mass-damper system is popular to be used as an analytical model for a cantilever based energy harvesting system for its simplicity in delivering a closed form solution. In this paper, a simple model of two piezoelectric energy harvesters is being developed to investigate the performance of a more generic multi-cantilever system. The electrical domain of both of the piezoelectric energy harvesters are modeled separately as an individual transformer. Spring-mass-damper model is used to model the mechanical domain of the piezoelectric cantilever under external mechanical excitation analogous to its equivalent electrical circuit consisting of resistor, inductor and capacitor. The analytical model of the mutli-cantilever is being validated with experiment results.**

Keywords* — *single-degree-of-freedom; broadband energy harvestin; piezoelectric cantilever

I. INTRODUCTION

A resonance based energy harvester in the form of cantilever is popular due to its simple structure while offering high output power density which is desirable for powering small electronic system compared to other energy harvesting mechanisms. However, due to the nature of ambient vibration souces with random and frequency varying pattern and also the physical limitation of miniature cantilever structure in harvesting optimum electrical power, therefore care is needed to design cantilever based energy harvester for practical application [1].

Cantilever based piezoelectric energy harvester has been studied as a single-degree-of-freedom spring-mass-damper system to estimate the electrical output when its vibrated to its resonance as described by Roundy S. *et al.* [2]. They modeled the piezoelectric energy harvester as a transformer equivalent circuit consisting of mechanical domain at the primary circuit and electrical domain at the secondary circuit. Essentially piezoelectric is a transformer converting power electromechanically through a vibrating piezoelectric structure or vice versa depends on whether the direct or converse piezoelectric effect is applied [3]. Rather than single-degree-of-freedom, Zhao and Erturk model a cantilevered bimorphs with distributed parameter [4] which has shown an improvement on the accuracy compared to experimental results for cantilever with no proof mass attached. Whereas for cantilever with proof mass attached, single-degree-of-freedom or lumped parameter models are still applicable [5].

In order to design optimum cantilever base energy harvester especially to harvest a vibration energy from ambient environment, an array of cantilevers arranged in a multi-cantilever system can effectively increase the bandwidth of energy harvesting [6]. Recently there were interests in investigate wider bandwidth energy harvesting using an array of piezoelectric cantilever connected to external circuitary [7, 8]. In this paper, an open-circuit model is being developed to investigate the performance of a multi-cantilever system in harvesting broadband vibration frequency without any external circuitry. A series of experiments were being conducted to validate the model.

II. EQUIVALENT CIRCUIT MODELING

An analytical model for the broadband energy harvesting device is essential in this study on the effect of polarity connections for each cantilevers. The equivalent circuit model can be use to observe the explicit relationships between the cantilevers connections to achieve energy harvesting at a broader spectrum of frequency. Furthermore, optimal system performance can be obtained through optimization of the geometrical properties of the cantilevers by using the circuit model. The analysis of the multiple cantilever beams energy harvesting system will be deduced from the approach of Roundy and Wright [2]. Fig. 1a shows the equivalent circuit configurations of the energy harvesting model with two cantilever beams.

The properties of the piezoelectric cantilever beam are first modeled into both mechanical and electrical portions as equivalent circuit elements. The electromechanical coupling between the mechanical and electrical circuits is then modeled using the properties of a transformer [3]. The equivalent input voltage which is the stress caused by the external input vibration for each cantilever beams is represented respectively by

$$\alpha_{in1} = k_{a1} m_1 \ddot{y} \quad \text{and} \quad \alpha_{in2} = k_{a2} m_2 \ddot{y} \tag{1}$$

where k_a is a geometric constant that relates the stress of the external input vibration force on the piezoelectric material, m_1 and m_2 are the proof mass for each cantilever repectively, and $\ddot{y}$ is the total input vibrations.

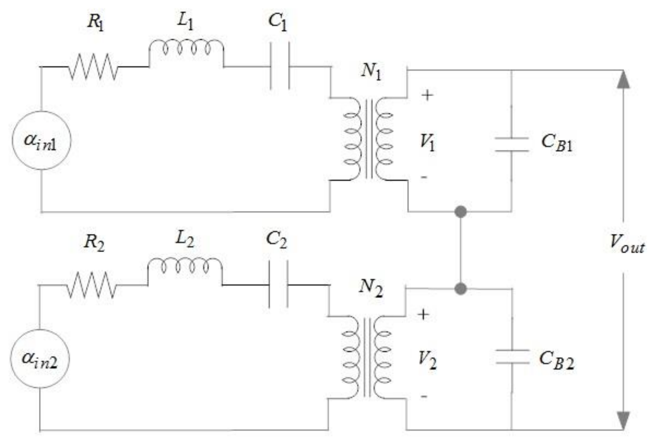

Fig. 1. Equivalent circuit configurations for two cantilever beams connected in series.

The equivalent resistor (R), inductor (L), and capacitor (C) in the mechanical portion of the circuit are characterize by the mechanical damping, mass, and mechanical stiffness of the energy harvester system repectively and can be written as

$$R_1 = k_{a1}k_{b1}b_{m1} \quad \text{and} \quad R_2 = k_{a2}k_{b2}b_{m2} \tag{2}$$

$$L_1 = k_{a1}k_{b1}m_1 \quad \text{and} \quad L_2 = k_{a2}k_{b2}m_2 \tag{3}$$

$$C_1 = \frac{1}{C_{e1}} \quad \text{and} \quad C_2 = \frac{1}{C_{e2}} \tag{4}$$

where k_b is a geometric constant that relates the stress of the inertial force on the piezoelectric material, b_m is the mechanical damping coeeficient, and C_e is the elastic constant of the piezoelectric material.

The electromechanical coupling is characterized by the equivalent turns ratio of the transformer and can be genereally expressed as,

$$N_1 = \frac{a_1 d_{31} C_{e1}}{2t_{p1}} \quad \text{and} \quad N_2 = \frac{a_2 d_{31} C_{e2}}{2t_{p2}} \tag{5}$$

where a is a constant determined by the internal wire connections of the piezoelectric material, d_{31} is the piezoelectric charge coefficients, and t_p is the thickness of the piezoelectric material in the cantilever beam respectively.

The capacitor (C_B) in the electrical domain of the circuit is the capacitance of the piezoelectric bender and is written as

$$C_{B1} = \frac{w_1 \varepsilon_1 a_1^2 l_{e1}}{2t_{p1}} \quad \text{and} \quad C_{B2} = \frac{w_2 \varepsilon_2 a_2^2 l_{e2}}{2t_{p2}} \tag{6}$$

where w is the width of the cantilever beam, ε is the dielectric constant of the piezoelectric evaluated at constant stress, and l_e is the length of the electrode covering the piezoelectric material on the cantilever beam respectively.

Applying Kirchoff's law to the equivalent circuit for each cantilever beam, the sum of the voltages for the first cantilever beam can be express as [2],

$$\alpha_{in1} = R_1\dot{S} + L_1\ddot{S} + \frac{S}{C_1} + N_1V_1 \tag{7}$$

$$i_1 = C_{B1}\dot{V}_1 \tag{8}$$

Where S is the strain and i_1 is the current related to the strain and can be written as

$$i_1 = a_1 w_1 l_{e1} d_{31} C_{e1} \dot{S} \tag{9}$$

The equivalent model for the first cantilever can now be derived by substituting equation (7) and (8) with the respective variable to obtain the explicit relationship between strain and voltages and is represented by

$$\ddot{S} + \frac{b_{m1}}{m_1}\dot{S} + \frac{k_{s1}}{m_1}S = \frac{a_1 d_{31} k_{s1}}{2t_{p1}m_1} + \frac{\ddot{y}}{k_{b1}} \tag{10}$$

$$\dot{S} = \frac{a_1\varepsilon_1}{2t_{p1}d_{31}C_{e1}}\dot{V}_1 \tag{11}$$

where k_{s1} is the equivalent spring constant applied in the lumped spring mass model and is written as

$$k_{s1} = \frac{C_{e1}}{k_{a1}k_{a2}} \tag{12}$$

Taking the Laplace transform of both equation (10) and equation (11) and solving them simultaneously, the output voltage for the first cantilever can be expressed as

$$V_1 = \left[\frac{2t_{p1}d_{31}C_{e1}A_{in}}{a_1\varepsilon_1 k_{b1}}\right]\left[s^2 + \frac{b_{m1}}{m_1}s + \frac{k_{s1}}{m_1} - \frac{d_{31}^2 k_{s1} C_{e1}}{\varepsilon_1 m_1}\right]^{-1} \tag{13}$$

where A_{in} is the Laplace transform of input vibration, $\ddot{y}$ .By using the substitution of the following equations,

$$\frac{k_{s1}}{m_1} = \omega_{n1}^2 \ ; \ \frac{b_{m1}}{m_1} = 2\varsigma_1\omega_{n1} \ ; \ \frac{d_{31}^2 C_{e1}}{\varepsilon_1} = k_{31}^2 \ ; \ s = j\omega$$

where ω_{n1} is the resonant frequency of the first cantilever beam, ς_1 is the damping ratio applied in the lumped spring mass model, and k_{31} is the piezoelectric coupling coeeficient, equation (13) can be rewritten as equation (14).

$$V_1 = \left[\frac{2t_{p1}d_{31}C_{e1}A_{in}}{a_1\varepsilon_1 k_{b1}}\right]\left[j2\varsigma_1\omega\omega_{n1} + \omega_{n1}^2\left(1-k_{31}^2\right) - \omega^2\right]^{-1} \qquad (14)$$

$$V_{out} = \left[\frac{2t_p d_{31}C_e A_{in}}{a\varepsilon k_b}\right] \times \left[\frac{j2\varsigma\omega\left(\omega_{n1}+\omega_{n2}\right) + \left(1-k_{31}^2\right)\left(\omega_{n1}^2+\omega_{n2}^2\right) - 2\omega^2}{\left[\omega_{n1}^2\left(1-k_{31}^2\right)-\omega^2\right]\left[\omega_{n2}^2\left(1-k_{31}^2\right)-\omega^2\right] - 4\varsigma^2\omega^2\omega_{n1}\omega_{n2} + j2\varsigma\omega\left[\omega_{n1}\left(\omega_{n2}^2\left(1-k_{31}^2\right)-\omega^2\right) + \omega_{n2}\left(\omega_{n1}^2\left(1-k_{31}^2\right)-\omega^2\right)\right]}\right] \qquad (15)$$

$$V_{out}Alt = \left[\frac{2t_p d_{31}C_e A_{in}}{a\varepsilon k_b}\right] \times \left[\frac{j2\varsigma\omega\left(\omega_{n2}-\omega_{n1}\right) + \left(1-k_{31}^2\right)\left(\omega_{n2}^2-\omega_{n1}^2\right)}{\left[\omega_{n1}^2\left(1-k_{31}^2\right)-\omega^2\right]\left[\omega_{n2}^2\left(1-k_{31}^2\right)-\omega^2\right] - 4\varsigma^2\omega^2\omega_{n1}\omega_{n2} + j2\varsigma\omega\left[\omega_{n1}\left(\omega_{n2}^2\left(1-k_{31}^2\right)-\omega^2\right) + \omega_{n2}\left(\omega_{n1}^2\left(1-k_{31}^2\right)-\omega^2\right)\right]}\right] \qquad (16)$$

Similarly, the same set of equations can be obtained for the second cantilever beam to solve for the output voltage, V_2.

In this paper, each piezoelectric cantilever beams is assumed to have the same geometrical properties and the resonant frequency is differed by adjusting the proof mass at the end of cantilever beams respectively. Thus, the total voltage of the equivalent circuit model is found by taking the sum of the voltages for both cantilever beams and is further simplified as equation (15). Furthermore, two cases of polarity connections for the cantilever beams are investigated in this paper. The first setup involves the series connection of two cantilever beam with the same polarity as shown in Fig. 1. For the second case, the cantilever beams are connected in series but with alternating polarity as illustrated in Fig. 2. Due to the change of voltage phase and polarity, the total output voltage can be further deduced as equation (16).

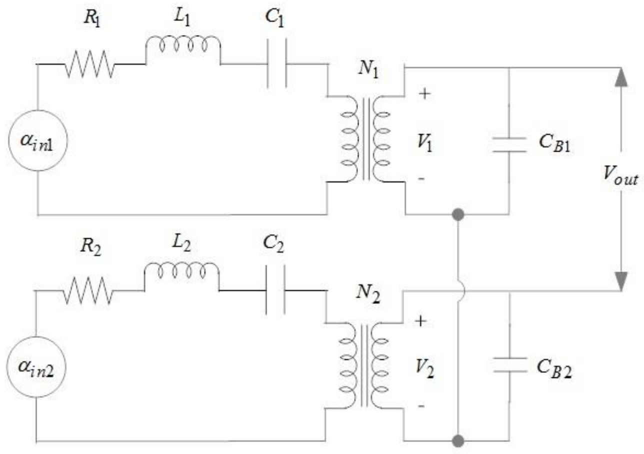

Fig. 2. Equivalent circuit configurations for series connection of two cantilever beams with alternating polarity.

Fig. 3 shows the plot of the analytical model. The bandwidth of the energy harvester is depended on the distance of individual resonant frequency and the output voltage in between the resonant frequencies is depended on the terminal polarity connecting the two piezoelectric cantilevers. The higher the output voltage at the center that overlapping between the two adjacent resonat frequencies the better in generating electrical output within the bandwidth which can be achieved by connecting the terminal of the piezoelectric in series with alternating polarity.

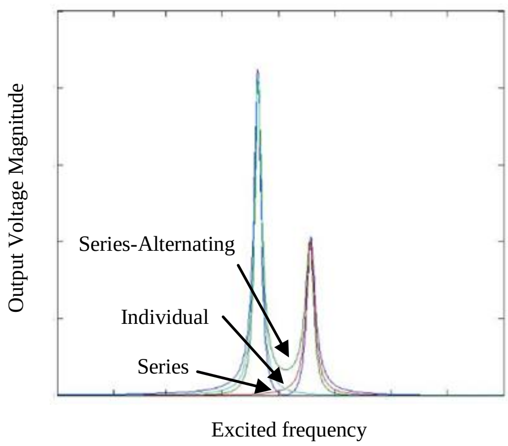

Fig. 3. Plotting of analytical model for two cantilevers with different resonant frequencies when connected in series and series-alternating configurations.

III. Experimental Set-up

The piezoelectric cantilevers which were being used in this experiment are a standard quick-mount bending generators with pre-mounted and wired at one end (Q220-A4-103YB) from Piezo Systems, Inc. The piezoelectric energy harvesters operate in bending mode in the form of a cantilever, whereby electrical charges develop when one side of the piezoelectric layer is stretched and the other side is compressed. These charges are collected at the terminals of the electrodes and measured as an open-circuit output voltage. The dimension of the piezoelectric generator and its performance is shown in Fig. 4.

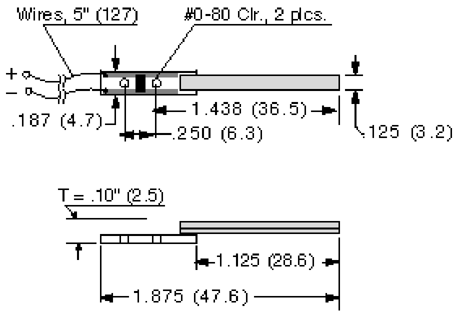

Fig. 4. Dimensions of the piezoelectric generator Q220-A4-103YB used in this experimental

In order to validate the analytical model, a series of identical cantilevers are mounted together as a multi-cantilever system on an electrodynamics shaker as shown in Fig. 5. The resonant frequency of the multi-cantilever is adjusted by placing a plasticine as the proof mass with different weights on the tip of the cantilever. An increment of 0.05 g of proof mass is used as denoted as C2, C3 and C4 with proof mass of 0.05 g, 0.1 g and 0.15 g respectively. C1 is a reference cantilever with no proof mass attached. Throughout the experiment, the multi-cantilever is excited to frequency up to 500 Hz at constant acceleration level of 1g (9.81 m/s^2).

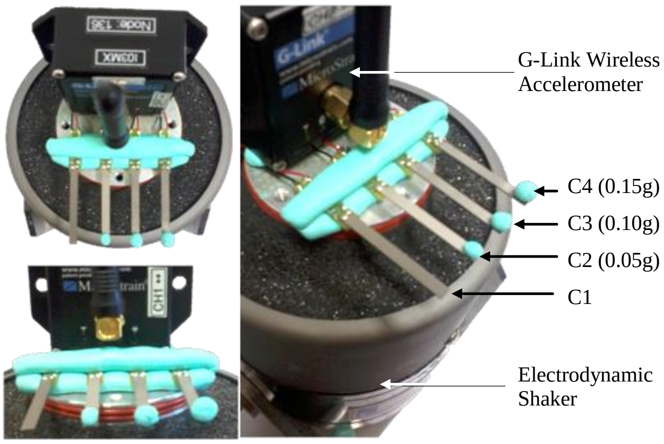

Fig. 5. Experimental Set-up of mult-cantilever system with variation of proof masses.

IV. Experimental Result

From the experimental results as shown in Fig. 6, the resonant frequency of the cantilever is reduced as proof mass is added, as expected. The distinctive fundamental frequency response of the cantilever shows that the plasticine is effectively acting as a proof mass in tuning down the resonant frequency of the cantilevers. It is also noticed that the open-circuit output voltage in RMS increases from 2.7 V to 5.4 V when a proof mass of 0.05 g is added, however, it drops when more proof mass is added as shown by C3 and C4 due to overdamping effect on the structure.

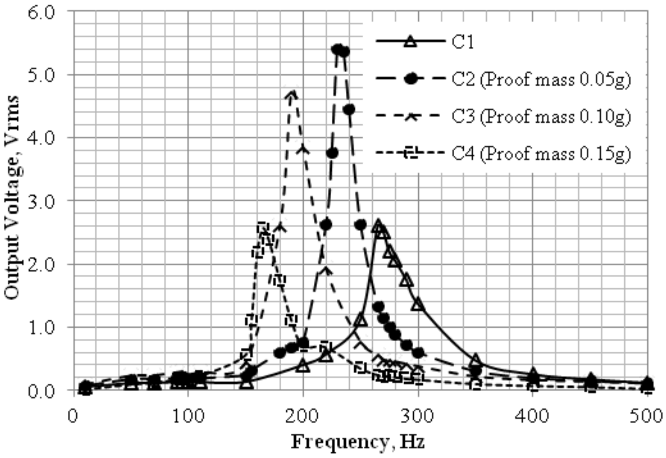

Fig. 6. Frequency response of the cantilevers with proof masses (0.05g, 0.10g, and 0.15g) and no proof mass at a constant acceleration level of 1-g (9.81 m/s2).

After the resonance frequency of the piezoelectric cantilevers are tuned to different values, the cantilevers are then connected in two different connections in order to identify the most optimum connection to harvest energy from a wide frequency band. The piezoelectric cantilevers are first connected in regular series connecting both of the piezoelectric cantilever as an AC source subsequently similar to the equivalent cicuit shown in Fig. 1. The open-circuit output voltage of the connection is then measured. This is followed by connecting the piezoelectric cantilever in series but with alternating polarity siminar to the equivalent circuit as shown in Fig. 2.

Fig. 7 shows the experimental results for regular series connected configuration when two, three and four piezoelectric cantilevers with different resonant frequencies are connected together as a multi-cantilever system. It is obviously shown that the output voltage in between two overlapping resonant frequencies is at its minimum which is lower compared to individual performance shwon in Fig. 6, when they are not connected. An improvement on the level of overlapping output voltages can be seen in Fig. 8, when the piezoelectric cantilevers are connected in series but with alternating polarity which is in agreement to that predicted by the anlaytical model.

V. Conclusion

The bandwidth of a piezoelectric based energy harvester can be effectively improved by using a multi-cantilever system consists of an array of piezoelectric cantilever with different resonant frequencies. The total bandwidth is depends on the number of the piezoelectric cantilever and the distance between the lowest and highest resonant frequencies of the system. However, as the gap between two adjacent resonant frequencies become wider the overlapping output voltage would be minimized lead to a poor overall performance. The analytical model predict that when the cantilevers are connected in series but with alternating polarity the overlapping output voltage can be improved which is verified by experimental results.

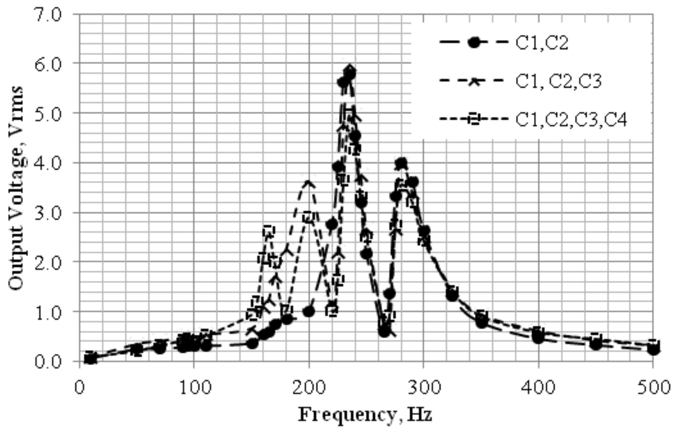

Fig. 7. Frequency response of the regular series connected configuration for two (C1 and C2), three (C1, C2 and C3) and four (C1, C2, C3 and C4) connected cantilevers when excited at a constant acceleration level of 1-g (9.81 m/s2).

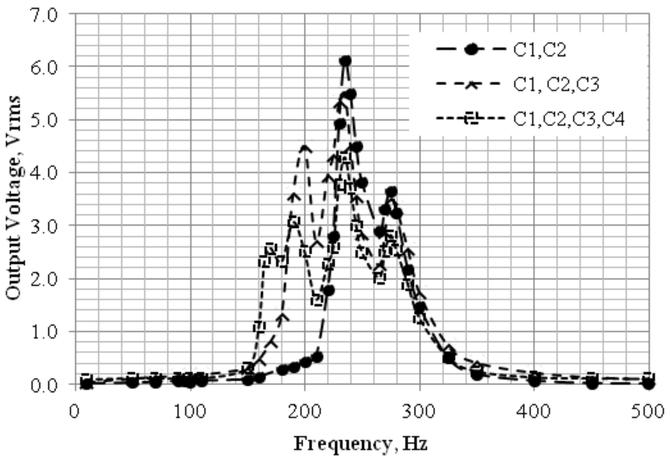

Fig. 8. Frequency response of the series-alternating connected configuration for two (C1 and C2), three (C1, C2 and C3) and four (C1, C2, C3 and C4) connected cantilevers when excited at a constant acceleration level of 1-g (9.81 m/s2).

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