Friction Compensation for Precise Positioning System using Friction-Model Based Approach

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Abstract—Friction force is an undesirable and nonlinear disturbance force that greatly influenced the accuracy and precision performance of machine tools. The main effect of friction forces is the formation of quadrant glitches during motion reversal. The traditional linear controller such as proportional-integral-derivative (PID) controller is required to be extended to model or non-model based techniques to provide high tracking performances. This paper mainly focuses on friction compensation of ball-screw driven system using nonlinear PID (N-PID) controller and friction model-based techniques. Adaptive behavior is introduced into PID controller by adding a nonlinear function. Two friction models namely; static friction model and Generalized Maxwell-slip (GMS) model are used. Experimental results showed that static friction model feedforward produced superior results in tracking error reduction whereas GMS model produced greater results in reduction of quadrant glitches magnitude. A combined model feedforward with N-PID controller that consists of both static and GMS friction models showed best performance in terms of tracking error and magnitude of quadrant glitches reduction as both advantages of respective individual friction models are combined and complement to each other.

Keywords—accuracy; friction compensation; friction model; machine tools; precision

I. INTRODUCTION

Friction is one of the undesired disturbance forces that affected the tracking accuracy and precision of machine tools. One of the typical effects of friction forces is the formation of quadrant glitches or “spikes” during velocity reversal, i.e. the circular motion [1-2]. Thus, the compensation of this disturbance force is highly desired.

Many friction compensation methods had been recorded and reviewed throughout the decade [3-4] and these methods are basically divided into two categories, namely: friction-model based and friction-model free approaches. In friction-model based approach, friction models are used to characterize the friction behavior and cancelled out the frictional effect in the system. Many friction models had been introduced and validated experimentally in the literature such as the static friction model [5], LuGre model [6] and Generalized Maxwell-slip (GMS) model [7]. Recently, some extended versions of GMS model are introduced in literature such as Smoothed GMS (S-GMS) model [8] and Modified GMS (M-GMS) model [9]. These friction models described the characteristics of friction forces in pre-sliding and sliding regime that dominantly dependent on displacement and velocity respectively. Feedforward of these friction models into the system had been proven to reduce the frictional effect of system and example of such records can be found in [10].

On the other hand, friction-model free approach compensated friction by utilizing the mathematical equation or iteration method, that is, through the application of linear or nonlinear controllers. Typical examples of linear controllers are inverse-model-based disturbance observer [10] and repetitive controller (RC) [11] while sliding mode controller (SMC) [12] and nonlinear proportional-integral-derivative (N-PID) controller [13] are examples of nonlinear controllers. However, RC is normally applied for system with periodic disturbances while the high cost and complexity in application becomes a great challenge in applying SMC. Typical option of application in industry is the disturbance observer and N-PID controller. N-PID controller is attractive due to its simplicity, complementary and ease of application into the existing system that typically using PID or cascade controller. In addition, the nonlinear function in N-PID controller also introduced adaptive property into the system.

This paper proposes a simple and efficient control approach for friction compensation using both friction-model based and friction-model free techniques. Friction is compensated using the feedforward of static friction model, GMS model or the combination of both friction models, in combination with N-PID controller as base controller.

This paper is organized as follows. Section II describes the experimental setup and Section III discusses the structure and identification of parameters for the two friction models applied. Section IV shows the design of controller as well as the overall control scheme for friction compensation. Section V compares and discusses the results based on the tracking performance in terms of tracking error and lastly Section VI summarizes and concludes the findings of this paper.
II. EXPERIMENTAL SETUP

The considered test setup is a ball-screw driven XYZ Stage that consists of a two axes milling table, namely; x-axis and y-axis as shown in Fig. 1. These axes are driven by a Panasonic MSMD 022G1U AC servomotor and equipped with an incremental encoder with resolution of 0.0005 millimeter/pulse respectively. Fig. 2 shows the schematic diagram of overall experimental setup.

![Image of XYZ Stage](image)

**Fig. 1. XYZ Stage used in experimental setup.**

![Image of Schematic Diagram](image)

**Fig. 2. Schematic diagram of overall experimental setup.**

The milling table is powered by servo amplifier and is connected to the dSPACE DS1104 Digital Signal Processor (DSP) board. This DSP board is linked with personal computer that equipped with Matlab and ControlDesk software. The main function of the DSP board is acted as data acquisition unit to send, collect and receive data between computer and the milling table. Only x-axis is considered in this paper.

The dynamics of the considered system is described using single-input-single-output (SISO) model and it is estimated using frequency domain identification approach. By utilizing the band-limited random excitation signal, the input and output measurements are obtained and these data is estimated using H1 estimator [14] to produce the SISO frequency response function (FRF) as shown in Fig. 3. Parametric model is fit on the FRF by using nonlinear least square frequency domain identification method [14] and thus, produced a second order transfer function with delay of 0.00129 seconds as shown in (1).

\[
Y(s) = \frac{A}{U(s)} = \frac{1}{s^2 + Bs + C},
\]

with \( A = 19.85 \text{ mm/V s}^2, B = 168.4 \text{ V/s} \) and \( C = 176.8 \text{ V s}^2 \), where \( Y \) represents the output position in unit of millimeter while \( U \) represents the input voltage to the drive in unit of volt.

![Image of Bode Diagram](image)

**Fig. 3. FRF measurement and estimated model of x-axis.**

III. FRICTION MODEL IDENTIFICATION

Two friction models, namely; static friction model and GMS model are considered and identified in this paper.

A. Static Friction Model

Static friction model is a well-known model that characterizes the friction behaviour in sliding regime, i.e. dependent on sliding velocity, \( v \). This model incorporates Coulomb, Stribeck and viscous friction yielding (2).

\[
F(v) = F_c + (F_s - F_c) \cdot \exp \left( -\frac{v}{V_s} \right) + \sigma \cdot |v| \cdot \text{sign}(v)
\]

\( F_c, F_s, \) and \( \sigma \) indicate Coulomb, static and viscous friction forces respectively. \( \delta \) represents the Stribeck shape factor while the Stribeck velocity is represented by \( V_s \). The identification method for this model is described in [15] and the identified parameters are tabulated in Table I.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( F_c ) (N)</th>
<th>( F_s ) (N)</th>
<th>( V_s ) (mm/s)</th>
<th>( \sigma ) (N s/mm)</th>
<th>( \delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>0.50</td>
<td>3.80</td>
<td>6.67</td>
<td>6.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

B. GMS Model

GMS model describes the friction behavior in both sliding and pre-sliding regime based on the Maxwell-slip model that consists of \( N \) different elementary slip-blocks and springs in parallel connection. This model incorporates the Stribeck curve during constant velocity, frictional memory in sliding regime...
and the non-local memory of hysteresis function in pre-sliding regime yielding (3) and (4).

\[
\frac{dF_i}{dt} = k_i v \quad (3)
\]

\[
\frac{dF_i}{dt} = \text{sign}(v) \cdot C \left( \alpha_i \cdot \frac{F_i}{s(v)} \right) \quad (4)
\]

During sticking process, i.e. velocity reversal, friction acted as a spring model with stiffness, \( k_i \). When the elementary friction force, \( F_i \), equals to a maximum value of Coulomb force, \( W_i = \alpha_i s(v) \), slipping process occurred. \( \alpha_i \) represents the normalized sustainable maximum friction force of each element during sticking and \( s(v) \) represents the Stribeck curve. \( C \) (equals to \( 1/V_i \)) is a constant parameter that indicates the rate of friction force followed the Stribeck effect in sliding. The summation of output of all elementary models and viscous constant \( \sigma \) (if presence) yielding (5) that denotes the total friction force.

\[
F_f(t) = \sum_{i=1}^{N} F_i(t) + \sigma \cdot v(t) \quad (5)
\]

In this paper, a GMS model with four elementary slip-blocks \( (N = 4) \) is selected. A virgin curve is formed based on the measurements at few different constant velocities to identify the friction characteristics during pre-sliding regime. Details of identification approach are recorded in [15] and the identified parameters for the model are tabulated in Table II.

<table>
<thead>
<tr>
<th>TABLE II. PARAMETERS OF GMS MODEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_i )</td>
</tr>
<tr>
<td>( \alpha_1 )</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
</tr>
<tr>
<td>( \alpha_3 )</td>
</tr>
<tr>
<td>( \alpha_4 )</td>
</tr>
</tbody>
</table>

### IV. DESIGN OF CONTROLLER

This section discusses the design of controller, i.e. the N-PID controller. N-PID controller is one of the extensions of conventional PID controller where a selected nonlinear function is added into the PID controller in cascade form. Addition of the nonlinear function allowed the controller to perform adaptively. A PID controller is designed prior to the selection of nonlinear gain function. In this paper, the PID controller is designed using traditional loop shaping method in frequency domain [16]. The parameters of the designed PID controller are tabulated in Table III.

<table>
<thead>
<tr>
<th>TABLE III. PARAMETERS OF PID CONTROLLER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
</tr>
</tbody>
</table>

| Values | 1.2051 | 1.2051e-3 | 6.0257e-3 |

Fig. 4, Fig. 5 and Fig. 6 shows the Bode plots, Nyquist plot and sensitivity function of the designed PID controller respectively. The properties of the designed PID controller are tabulated in Table IV.

<table>
<thead>
<tr>
<th>TABLE IV. PROPERTIES OF PID CONTROLLER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Properties</td>
</tr>
<tr>
<td>Gain margin (dB)</td>
</tr>
<tr>
<td>Phase margin (degree)</td>
</tr>
<tr>
<td>Bandwidth (Hz)</td>
</tr>
</tbody>
</table>
There are many choices of established nonlinear functions can be found in literature. The selected sector bounded nonlinear gain function for this paper is shown in (6) and (7) due to its simplicity [17].

\[
k(e) = \frac{\exp(k_0 e) + \exp(-k_0 e)}{2}; \quad e = \begin{cases} e \leq e_{\text{max}} & e_{\text{sign}}(e) \leq e_{\text{max}} \\ e > e_{\text{max}} & \end{cases}
\]

Equation (6) derives the nonlinear gain as a function of error, \( e(t) \) which bounded in \( 0 \leq k(e) \leq k(e_{\text{max}}) \). Positive constant, \( k_0 \) represents the rate of variation of nonlinear gain while \( e_{\text{max}} \) denotes the range of variation of error in the unit of millimeter. Scaled error is represented as \( f(e) \) in (7). The overall equation of N-PID controller is shown in (8).

\[
G_{\text{N-PID}}(s) = k(e) \times (K_p + \frac{K_i}{s} + K_d s)
\]

Two parameters are required to be tuned in for N-PID controller, i.e. \( k_0 \) and \( e_{\text{max}} \). These two parameters are tuned using heuristic method based on the Popov stability criterion and the details of design procedure are discussed in previous research [18]. A Popov plot is plotted using Matlab coding as shown in Fig. 7 and this plot is used to determine the maximum value of \( k(e) \).

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\]
Fig. 10. Measured tracking errors for (a) no friction feedforward, (b) static friction model feedforward, (c) GMS model feedforward, and (d) feedforward with combining models.

### TABLE V REDUCTION OF TRACKING ERROR

<table>
<thead>
<tr>
<th>Friction compensation method</th>
<th>Tracking error (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No feedforward</td>
<td>0.0191</td>
</tr>
<tr>
<td>Static friction model</td>
<td>0.0012</td>
</tr>
<tr>
<td>GMS model</td>
<td>0.0188</td>
</tr>
<tr>
<td>Static + GMS model</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

### TABLE V REDUCTION OF QUADRANT GLITCH

<table>
<thead>
<tr>
<th>Friction compensation method</th>
<th>Quadrant glitch (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No feedforward</td>
<td>0.0050</td>
</tr>
<tr>
<td>Static friction model</td>
<td>0.0040</td>
</tr>
<tr>
<td>GMS model</td>
<td>0.0035</td>
</tr>
<tr>
<td>Static + GMS model</td>
<td>0.0025</td>
</tr>
</tbody>
</table>

Based on the results in Fig. 10, Table IV and Table V, it is clearly shown that the friction model feedforward approach is effective in friction compensation. Static friction model feedforward greatly reduced the overall tracking errors while only slight reduction of tracking errors is observed for GMS model feedforward. This phenomenon is observed because static friction model is highly focused on sliding regime of friction that dependent on velocity and Stribeck effect. Hence, static friction model contributed more on the reduction of tracking error along the motion. Conversely, GMS model is focused more on the reduction of friction in pre-sliding regime, i.e. quadrant glitches and therefore, its effect in reduction of tracking error is limited.

On the other hand, GMS model provided better solution in reduction of quadrant glitches as it accounted the non-local hysteresis of friction in pre-sliding regime. However, the friction compensation component of GMS model in sliding regime is not as efficient as static friction model that specifically targeted on friction in sliding regime. Thus, GMS model is more pronounced in reduction of quadrant glitches in this case. In the case of feedforward with both friction models, promising results are obtained where tracking errors and quadrant glitches are greatly reduced. These results showed that both friction models are complemented to each other and both of their advantages are utilized.

### VI. CONCLUSION

Friction model feedforward approach is applied for friction compensation of a ball-screw driven system in this paper. Static friction model and GMS model are selected to characterize the friction forces and N-PID controller is used as the base controller in the system. Results showed that the feedforward of both friction models simultaneously provided the superior results in terms of tracking errors and quadrant glitches reduction. Static friction model is excellent in tracking errors reduction while GMS model is pronounced in quadrant glitches reduction. These benefits of both friction models are combined as both models are complement to each other when both models are feedforward together.

### ACKNOWLEDGMENT

This research is financially supported by Centre for Research and Innovative Management (CRIM) of Universiti Teknikal Malaysia Melaka (UTeM). Authors also would like to thank Faculty of Manufacturing Engineering in sponsoring the equipment and laboratory.

### REFERENCES


